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Debbie Cameron

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IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

BEFORE THE BOARD OF PATENT APPEALS AND INTERFERENCES

In re Application of: **John O. Lamping et al.**

Application No. **09/124,805**

Confirmation No. **7115**

Filed: **29 July 1998**

Title: **LOCAL RELATIVE LAYOUT OF NODE-LINK STRUCTURES IN SPACE WITH NEGATIVE CURVATURE**

Group Art Unit: **2672**

Examiner: **WANG, JIN-CHENG**

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APPEAL BRIEF

Sir:

This Appeal Brief is filed in support of Appellants' appeal from the Office Action mailed October 6, 2004, in this case. A Notice of Appeal was mailed to the Patent and Trademark Office on April 6, 2005.

The appropriate fee as set forth in § 41.20 (b)(2) of **$500.00** is covered in the attached Credit Card Payment Form (PTO-2038). Also submitted herewith is a Request for Extension of Time, together with the appropriate fee. Should it be determined that additional fees are required, the Commissioner is hereby authorized to charge those fees or credit any overpayment to Deposit Account No. 50-0869 (File No. INXT 1002-1).

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I. REAL PARTY IN INTEREST

The real party in interest is **Inxight Software, Inc.**, the assignee of record.

II. RELATED APPEALS AND INTERFERENCES

There are no known appeals or interferences relating to this case.

III. STATUS OF CLAIMS

Claims 1-16 were canceled.

Claims 17-44 are pending in this case and all have been rejected. All of the rejected claims are being appealed.

IV. STATUS OF AMENDMENTS

An amendment (Response F Under 37 C.F.R. §1.116) was filed subsequent to the Final Office Action, but the Examiner declined to enter it.

V. SUMMARY OF CLAIMED SUBJECT MATTER

1. Overview of the Invention

The invention pertains to ways of displaying very large node-link structures, such as hierarchical tree structures. Often these are very difficult to display satisfactorily, because as the number of levels in the tree increases, the amount of information to be displayed tends to increase exponentially. Very quickly, there is not enough space on a display screen to show it all.

The present invention provides an improvement in a solution to this problem described in the inventors' own earlier patent, Lamping U.S. Patent No. 5,619,632 ("Lamping '632").[1] In Lamping '632, roughly described, the node-link structure is displayed in what appears to be a hyperbolic plane, with the root node at the center of a disk and the branches extending outwardly from the root node toward the edges of the disk. For example, see Fig. 15 of Lamping '632, reproduced below.

A user can interactively click and drag the image around on the disk, and as elements of the structure are moved closer to an edge of the disk, they get smaller and smaller until they disappear altogether. Meanwhile, elements toward the opposite edge of the disk become larger and larger, making room for more detail and/or further child elements of the node-link structure to become visible. See, for example, Fig. 16 of Lamping '632, also reproduced below.

[1] Lamping '632 has overlapping but not identical inventorship with the present application. In this paper and all others submitted in the context of the present application, Appellants may discuss *teachings* of the earlier Lamping patents. No statement made in these papers should be considered as commenting on the scope of any *claims* of the earlier Lamping patents.

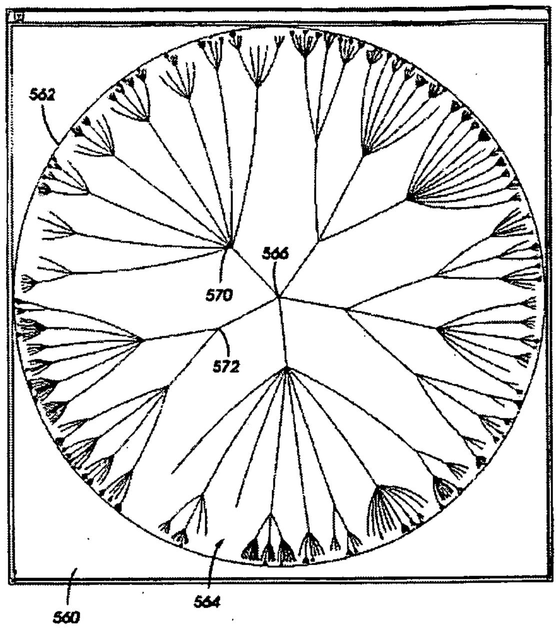


FIG. 15

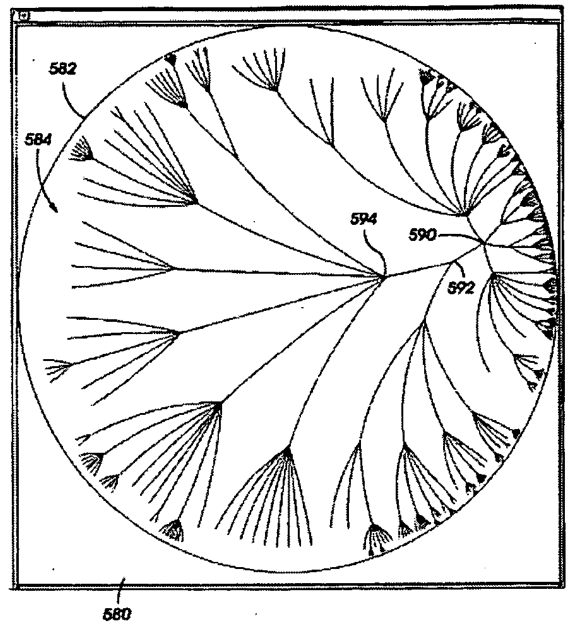


FIG. 16

Other examples of this can be seen, for example, in the sequences of Figs. 20 and 21 of Lamping '632.

Thus by dragging the image around within the disk on the display, a user can increase or decrease the detail visible in desired parts of the node-link structure, thereby mitigating the problem of displaying large node-link structures in a limited amount of display space. The arrangement allows the user to see large portions of the node-link structure for context, and at the same time to focus in on the detail in any particular part of the structure as desired.

Working examples of node-link structures displayed in this manner can be found at any of the following web sites:

1. http://www.inxight.com/products/sdks/st/

2. http://nsdl.org/collection/ataglance/browseBySubject.html

3. http://www.flashkit.com/search/sitemap/index.shtml

The techniques in Lamping '632 make the node-link structures <u>*appear on the display* as if they were in a hyperbolic plane, but of course they are not. They are displayed on an ordinary computer monitor, which is flat</u>.

Lamping '632 achieves this by a two-step process.

- First, the node-link structure is "laid out" in a "layout space", to obtain "layout data", indicating "positions" for parts of the node-link structure in the layout space.
- Second, the system uses the layout data to obtain "mapped data" for a representation of the node-link structure on a disk-shaped region on a flat display.

Lamping '632, col. 16, lines 53-62; Lamping '632, Fig. 8 steps 304-306, and corresponding text at col. 19, lines 7-10 (among other places).

The invention of the present application concerns improvements in the first of these steps (laying out the node-link structure in a space with negative curvature), not the second of these steps (mapping the element positions from the layout space to flat display space).

2. Brief Explanation of Spaces with Negative Curvature

Because the Examiner seems not to understand the concept of a space with negative curvature, it is believed worthwhile to briefly explain it here. A space with negative curvature is a mathematical concept of a space having certain well-established properties, one of which is that parallel lines diverge, and another of which is that "through any position in a space with negative curvature that is not on a given straight line, there are multiple other straight lines parallel to the given straight line." Specification, p. 11, lines 3-6.

These are very commonly used definitions for such a space, used in numerous mathematics textbooks and publications in addition to Appellants' specification. See, for example, "Space Properties", pp. 1-6 (2002), available at http://scholar.uwinnipeg.ca/courses

/38/4500.6-001 /Cosmology/Properties_of_Space.pdf, visited 6/4/2004, p. 5 of 11, submitted with Appellants' Response on July 13, 2004 and attached as an Exhibit hereto:

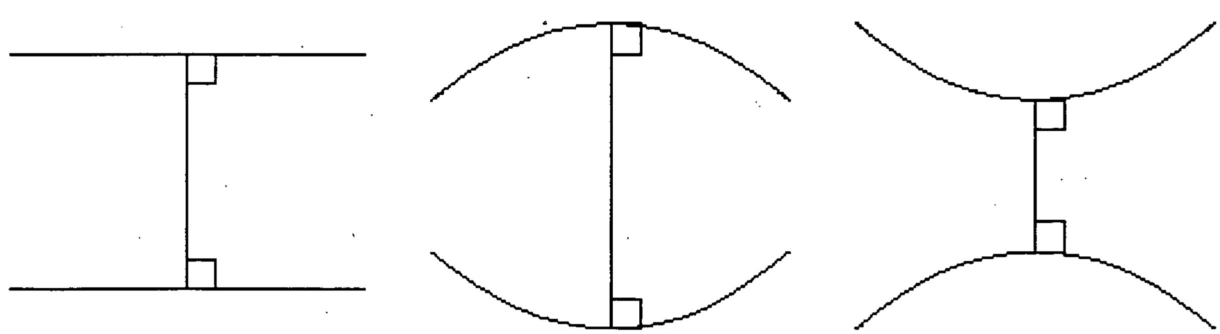
> "In Euclid's geometry all surfaces are flat and parallel lines always stay the same distance apart, never meeting and never diverging. However, in curved non-Euclidean geometries, lines that start off parallel eventually cross each other in the positive curvature case, while these same lines diverge from each other in the negative curvature case."

See also Anderson, "Hyperbolic Geometry", p. 7, submitted with the IDS on April 6, 2005 and attached as an Exhibit hereto:

> "Hyperbolic geometry is then defined by using the same set of axioms as Euclidean geometry, with the hyperbolic variant of the Parallel Postulate, namely that given a hyperbolic line L and a point P not on L, there are at least two hyperbolic lines through P and parallel to L.".

Various other textbooks might define the term by referring to other properties of a space with negative curvature, but those definitions can be derived mathematically from the ones set out herein, and vice-versa. That is, they all define a space having all the same properties.

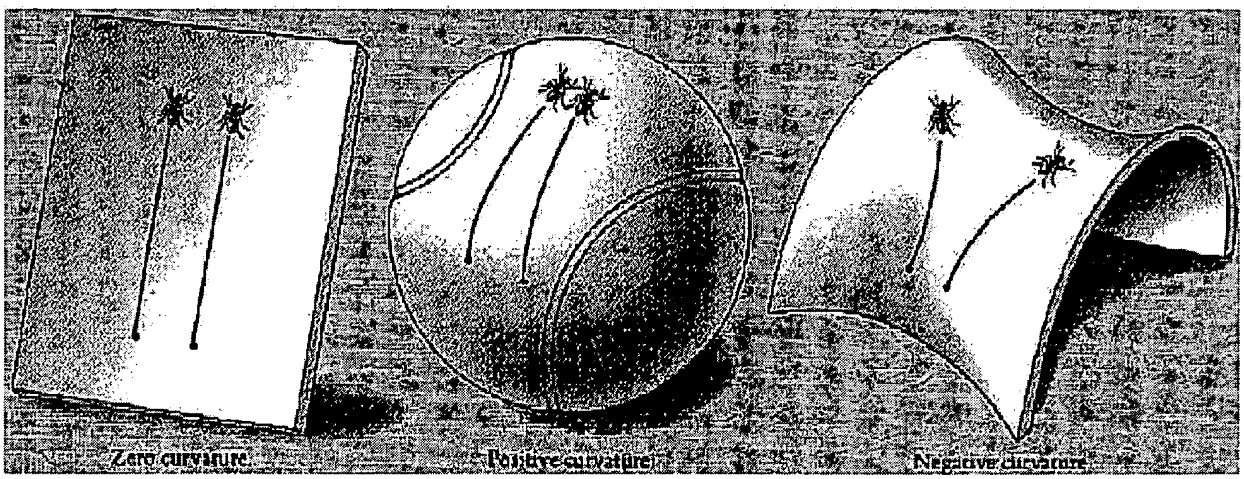
To visualize a space with negative curvature, one may consider the following three drawings:



Euclidean Elliptic Hyperbolic

In the normal, Euclidean (flat) space, with which we are all familiar, if two lines are parallel to each other because they both intersect a common vertical line at right angles, then the two lines always remain at the same distance apart. In Non-Euclidean spaces, on the other hand, the two lines can either diverge or converge. In a space with positive curvature, such as an elliptic space, the two parallel lines converge. In a space with negative curvature, such as a hyperbolic space, the two parallel lines diverge.

Another way to visualize the concept is to think of how the plane, on which the above lines are drawn, needs to be curved in order for the lines to seem straight. The following drawing, from the "Properties of Space" article cited above, illustrates this:



On a flat plane, a plane with zero curvature, if two ants start out traveling on two parallel lines, they will remain the same distance apart as long as they keep walking straight. But in a space with positive curvature (pictured in the middle), as the two ants continue to walk on what seems to them to be completely straight lines, they will in fact converge toward each other and eventually intersect. Similarly, in a space with negative curvature (pictured on the right), as the two ants continue to walk on what seems to them to be completely straight lines, they will in fact

diverge away from each other. The lines diverge not because the lines are curved, but because the surface on which they are drawn is curved.

In Lamping '632, it is the layout space with negative curvature (where parallel lines diverge), together with the mapping functions for projecting the positions of elements in the layout space onto the flat display space, which is so effective at showing the user a large node-link structure in such a way that allows the user to see large portions of the structure for context, but to focus in on particular desired portions where more detail is desired.

3. Deficiencies in the Prior Art

In the Lamping '632 methods, dragging the structure around on the display does not require a re-layout of the structure in layout space. The only thing that changes is the mapping of the positions in layout space to flat display space. But if a *structure* changes after it is displayed, for example if the user inserts or deletes an element, then a new layout into the layout space with negative curvature must typically be done for at least a large part of the structure as changed. Then the new mapping is performed from the new positions in layout space into the flat display space and redisplayed.

Performing layout on a large structure can be slow, and when taken together with redisplay, may prevent a user from effectively interacting with the changed structure. And unfortunately, many of the layout techniques described in Lamping '632 have certain characteristics that mandate layout of at least a large part of the structure. These layout techniques make decisions at each node that depend on layout decisions higher in the tree and at its sibling nodes; as a result, adding or deleting a child of a node mandates that the layout be redone for all descendants of the node and of any of its siblings.

Also, the layout techniques in Lamping '632 save only the node's absolute position in the hyperbolic plane; any change in the structure that would change a node's position requires repeating the layout of its siblings and all of its descendants. (Specification, page 3, line 7, through page 4, line 2.)

4. The Invention of the Present Application

Embodiments of the invention in the present patent application alleviate these problems and others by using a new technique for laying out elements of the node-link structure in the layout space with negative curvature. In particular, the new technique uses "nearby relationship data" (e.g. data from the node-link structure that describes the number of children that a parent node has, and/or the number of grandchildren that the parent node has) to derive "layout data" indicating what the element's position should be in the layout space. And the position of the element in layout space is not determined in absolute terms; it is determined and stored only in *relative* terms, *relative* to whatever position the parent has in layout space. (Specification, p.4).

For example, when an element, say a subject node, is to be laid out in layout space, it is first determined from the node-link structure how many other child nodes there are of the same parent node, and potentially also how many grandchild nodes there are of the same parent node. This "nearby relationship data" determines how much horizontal space to allocate for all the grandchild nodes in the layout space, how long the radial links will have to be in order to provide such horizontal space, what angles to provide between the radial links, and ultimately where the subject node will have to be positioned in the layout space in order to satisfy the positioning scheme. (Specification, pp. 4-5).

And the position is thus determined only in *relative* terms in the layout space, for example a distance ("position displacement data") in the layout space from its parent node and an angle in the layout space from its parent node ("angle displacement data", for example indicating an angular difference between an incoming link to the parent and the outgoing link from the parent to the subject element). (Specification, pp. 4-5) The position of the subject node is stored in layout space only in these terms (Specification, p. 5, lines 16-17); the absolute x,y position of the node in layout space does not need to be calculated and does not need to be stored. (Specification, p. 14, line 1, to p. 15, line 10 and Figs. 1 and 2).

Again, once the element positions are established in layout space, they are mapped to flat display space in accordance with the user's selection of what portion of the structure to display with greater detail. The mapping step can be similar to that described in earlier Lamping patents. (e.g., Specification, p. 22, lines 1-11, and boxes 314, 326 and 336 in Fig. 3).

The above techniques mean that when the node-link structure itself changes, only the local region around the change needs to be re-laid out in layout space. That is, it is no longer necessary to traverse the entire node-link structure, from the root node outward, in order to make the necessary changes in the layout space. Nor is it necessary to change any information about the location of child and further descendant elements, since positions in the layout space with negative curvature are stored only relative to their parents. Thus the implied absolute position in layout space of the subject element and all its descendants can be changed merely by making a small number of localized changes in the data structure. (Specification, page 4, line 3, through page 7, line 4.)

5. Concise Explanation of the Subject Matter Defined in Each Independent Claim (37 C.F.R. 41.37(c)(1)(v)

As mentioned in the Summary of Claimed Subject Matter above, the claims cover methods for *local relative* layout of a node-link structure in a space with negative curvature. Representative independent claim 17 focuses primarily on the *localness* of the layout methods, whereas representative independent claim 29 focuses primarily on the *relativeness* of the positions stored for the elements in the layout space.

Independent Claim 17 defines a method of laying out a node-link structure in a space with negative curvature. The claim calls initially for a step of obtaining "nearby relationship data" for a subject element. (Fig. 2, box 100.) The nearby relationship data is typically limited to relationships with elements only a short distance in the tree structure from the subject element in the node-link structure (see specification examples at p.5, lines 4-5; p.10, lines 1-7; p.13, lines 18-21; p.14, lines 1-8; p.32, lines 9-11; p.34, lines 4-7 and p.46, line 21 - p.47, line 5), but even if not, the claim specifically says that they cannot include all elements of the node-link structure ("the nearby relationship data excluding relationships with at least one element of the node-link structure").

Claim 17 then calls for a step of obtaining layout data identifying the subject element's position in a space with negative curvature. (Fig. 2, box 102.) The layout data has to be based only on "the" nearby relationship data (i.e. the nearby relationship data obtained in the step of obtaining), and therefore there must be at least one element of the node-link structure whose relationship with the subject element is not used in determining the layout data.

Independent Claim 29 also defines a method of laying out a plurality of elements of node-link structure in a space with negative curvature. Among other things, the claim states that after positions in the space with negative curvature for all elements in the plurality have been

calculated, the positions in the space with negative curvature of each element in the plurality are to be stored only relative to an element of the node-link structure other than a root element of the node-link structure. (Fig. 2, box 102; p.4, lines 7-9; p.5, lines 16-17; p.11, line 16 - p.12, line 2; p.14, lines 9-18; p.32, lines 12-14; p.34, lines 7-9, and especially p.37, lines 10-20.)

VI. GROUNDS OF REJECTION TO BE REVIEWED ON APPEAL

All pending claims 17-44 stand rejected under 35 USC 102(b) over Lamping U.S. Patent No. 5,619,632.

VII. GROUPING OF CLAIMS

Independent claims 17 and 26-28 and dependent claims 18 and 25 can be grouped together for common consideration in this appeal. Claim 17 will be considered representative of this group herein.

Independent claims 29 and 42-44 and dependent claims 30 and 37-41 can be grouped together for common consideration in this appeal. Claim 29 will be considered representative of this group herein.

All other claims are argued separately herein.

The above groupings are made only to simplify the issues for this appeal, and Appellants do not waive their right to argue all claims independently in any post-appeal proceedings.

VIII. ARGUMENT

As mentioned in the Summary of Claimed Subject Matter above, the claims cover methods for *local relative* layout of a node-link structure in a space with negative curvature. Representative independent claim 17 focuses primarily on the *localness* of the layout methods, whereas representative independent claim 29 focuses primarily on the *relativeness* of the positions stored for the elements in the layout space.

But whereas the claims cover methods for *laying out* the elements *into* a layout space with negative curvature, the Examiner's rejections of these claims all rely on methods taught in the Lamping '632 patent for *mapping* elements already positioned in the layout space with negative curvature, into the display space, which is *flat*. Not only do those techniques fail to anticipate Appellants' claims because they do not obtain or calculate layout data "identifying the subject element's position <u>in [a] space with negative curvature</u>", but they also fail to satisfy other limitations in anticipate Appellants' claims, including the obtaining of layout data based <u>only</u> on "nearby relationship data", and not on the position of any other element in the structure.

Appellants will discuss independent claim 17 and its dependent claims first, followed by independent claim 29 and its dependent claims.

1. Independent Claim 17, as Representative of Independent Claims 17 and 26-28 and Dependent Claims 18 and 25

Independent claim 17 calls for:

17. A method of laying out a node-link structure in a space with negative curvature; the method comprising:

obtaining nearby relationship data for a subject element in the structure, the nearby relationship data indicating information about nearby node-link relationships, the nearby relationship data <u>excluding relationships with at least one element</u> of the node-link structure; and

> based on only the nearby relationship data, and not on the position of any other element in the structure, obtaining layout data identifying the subject element's position in the space with negative curvature. (emphasis added).

Thus claim 17 calls for a step of obtaining *nearby* relationship data for a subject element. The *nearby* relationship data is typically limited to relationships with elements only a short distance from the subject element in the node-link structure (see specification examples), but even if not, the claim specifically says that they *cannot* include *all* elements of the node-link structure ("the nearby relationship data excluding relationships with at least one element of the node-link structure").

Claim 17 then calls for a step of obtaining layout data identifying the subject element's position in a space with negative curvature. The layout data has to be based only on "the" nearby relationship data (i.e. the nearby relationship data obtained in the step of obtaining), and therefore there must be at least one element of the node-link structure whose relationship with the subject element is not used in determining the layout data. Lamping '632 does not teach this feature at all.

In Lamping '632, the layout method in layout space (the space with negative curvature) always starts with the root node of the node-link structure, and always lays out the *entire* node-link structure, and always uses the relationship of *all* elements of the node-link structure in obtaining *each* element's position in layout space. See Lamping '632, Fig. 9 and col. 20, line 18 - col. 21, line 8. Lamping's layout routine is always called beginning at the root node (see step 350), and does not stop until it reaches all the way down to nodes that have no more children (step 360). For *each* element of the node-link structure which is to be laid out, the nearby relationship data for *all* other elements of the node-link structure are taken into account. There is

no "at least one element" of the node-link structure whose relationship with the subject element is *not* used in determining the layout data.

If Appellants understand the Examiner correctly, Lamping '632, Figs. 6-7 and col. 17, line 20 - col. 18, line 50, are cited for the proposition that the nearby relationship data used by Lamping to obtain layout data excludes relationships with at least one element of the node-link structure. (Final Office Action, p. 7, lines 4-7.) **But the cited parts of Lamping describe a method of mapping layout data *from* layout space to a disk in *flat display region*.** The layout space is a space with negative curvature, *but a display region is a* ***flat*** *space*. Lamping '632, col. 17, lines 28-33.

This is a transformation and mapping function that converts positions of the elements from layout space into the two-dimensional display space, which does not have negative curvature. It is <u>not a method of laying out a node-link structure in a space with negative curvature</u>, as called for in Appellants' claim 17. Nor is a step ever performed in these teachings of obtaining layout data in a space with negative curvature, <u>based on nearby relationship data</u>, also as called for in Appellants' claim 17. For these reasons alone, the cited teaching fails to anticipate and is not relevant to Appellants' claim 17.

The Examiner may consider Lamping '632, Figs 15-17, as showing that <u>display</u> space is a space with negative curvature. But what is shown in these drawings is a <u>flat</u> surface (e.g. a computer monitor) onto which the elements in the space with negative curvature have been mapped (projected).

From the drawings in the "Summary of Claimed Subject Matter" section above, one can perhaps see that when the two ant-traces on the saddle-shaped surface are projected onto a flat surface (for example onto the surface of the page on which the drawing is depicted), they will

appear curved. These ant traces are straight in the space with negative curvature, but curved in the projection onto a flat space.

So too with Figs. 15-17. These are projections onto a flat disk (Lamping '632, col. 24, lines 44-55) (e.g. displayed on a computer monitor) of lines which are straight in the layout space with negative curvature, and therefore curved in the flat display space shown.

The Examiner also cites Lamping '632 col. 21. line 11 to col. 25, line 23; col. 16, lines 45-63; col. 32, lines 19-35, col. 25, lines 52-62, col. 4 lines 44-50, and Figs. 5 and 17, but these excerpts, too, describe only the mapping to a flat display space or the movement of elements that a user would perceive on the flat display space as the mapping changes in an animation. In the case of Lamping '632 col. 16, lines 45-63, the excerpt only describes generally some of the overall steps performed by an implementation of the Lamping '632 methods, and does not teach a step of obtaining layout data "based on only the nearby relationship data, and not on the position of any other element in the structure."

In the Response to Arguments section of the Final Office Action, the Examiner appears to argue that an **arc** in Lamping '632's Fig. 15 constitutes a "space with negative curvature." Clearly this cannot be the case. As mentioned, a "space with negative curvature" however it is defined, must satisfy the properties that parallel lines diverge and that there are multiple other straight lines parallel to any given straight line in the space. (See Summary of Claims, above). *But no two parallel lines can exist in a mere arc.*

Remember that Fig. 15 of Lamping '632 is flat; it depicts a *projection* of a space with negative curvature, but it itself is flat. An arc on this flat surface therefore is simply a curved line. It has no width, so it cannot contain more than one line. And since it is curved, it cannot contain even one line, assuming the line is straight. It is simply way beyond the "broadest reasonable

interpretation" of the term "space with negative curvature" to assert that an arc in a flat plane can qualify.

The Examiner is correct that in Lamping's process of transforming element positions *from* layout space *to flat mapping space*, Lamping does avoid transforming elements that are too near the edge of the unit disk (Fig. 6, step 262). But no such exclusion is made in Lamping's process of laying out elements of the node-link structure *into* the space with negative curvature. In Lamping '632, as discussed above, the nearby relationship data of *all* elements of the node-link structure are used in obtaining *each* element's position in layout space.

Therefore the method cited by the Examiner:

(1) is not a method of laying out a node-link structure in a space with negative curvature,

(2) never performs any step of "obtaining layout data identifying the subject element's <u>position in the space with negative curvature</u>",

(3) never performs any step of obtaining the subject element's position based on <u>any</u> "relationship data", and

(4) *does* obtain position information based on the position of other elements in the space with negative curvature,

all of which are contrary to the limitations of Appellants' claim 17.

Appellants therefore respectfully submit that the Examiner has not made a *prima facie* case that claim 17 is unpatentable.

2. Dependent Claims 19-24

The Examiner rejected claims 18-25 initially in the Office Action mailed August 12, 2002, as being anticipated by Lamping '632. All references in this section to assertions made by the Examiner refer to that Office Action.

Appellants, in their Response C, filed February 12, 2003, provided numerous points explaining why claims 18-25 should be patentable in their own right. See Appellants' Response C, pp. 19-21. However, neither of the two subsequent Office Actions appear to address any of Appellants' points. They merely repeat the first-made rejections.

Claims 19-24 all depend ultimately from independent claim 17, and therefore should be patentable for at least the same reasons as claim 17. In addition, each of claims 19-24 adds further limitations which, it is submitted, render them patentable in their own right.

a. **Claim 19** ads a limitation that the layout data include position displacement data indicating a distance between the parent's position and the element's position, and:

> angle displacement data indicating an angular difference between an incoming link to the parent and an outgoing link from the parent to the element.

Nowhere does Lamping '632 teach such a feature.

The Examiner's only argument for rejection of claim 19 appears to be that set forth at p. 4, lines 17-20 of the August 12, 2002 Office Action, repeated verbatim in each of the three subsequent Office Actions:

> Re claims 2-4 and 18-20, Lamping discloses the space with negative curvature is a hyperbolic space (col.17, lines 28-44, col. 16, lines 53-62; col. 20, lines 20-52). Lamping teaches a negative curvature as a

hyperbolic space when he discloses the layout space is a hyperbolic plane.

But there is no argument here addressing the limitation of claim 19 calling for the layout data to include "angle displacement data indicating an angular difference between an incoming link to the parent and an outgoing link from the parent to the element". The Examiner therefore has failed to make *a prima facie* case that claim 19 of the unpatentability of claim 19.

Lamping '632 does speak of determining an angle at which to lay out elements in the space with negative curvature (for example at Lamping '632, col. 20, line 33), but does not say anything about that angle "indicating an angular difference between an incoming link to the parent and an outgoing link from the parent to the element" as called for in claim 19. The only way the Examiner can possibly believe that the mention of an angle in Lamping '632 is a teaching specifically of data "indicating an angular difference between an incoming link to the parent and an outgoing link from the parent to the element" is through the use of hindsight. Lamping '632 certainly does not suggest this, and the Examiner has not cited anything in Lamping '632 to support the Examiner's belief.

In fact, the angle referred to in Lamping '632 is defined only relative to a predetermined and fixed and absolute "zero direction". It does not in fact indicate "an angular difference between an incoming link to the parent and an outgoing link from the parent to the element" as called for in claim 19. This can be seen from a different patent, Lamping U.S. Patent No. 5,590,250 ("Lamping '250") , which Lamping '632 incorporates by reference (Lamping '632, col. 18, lines 8-15) for its teaching of the Layout step in Lamping '632. Lamping '250 says that the angles are specified relative to the "rightward" direction: "the zero direction from which other directions are measured." (Lamping '250, col. 23, lines 64-65). Clearly the angles referred to in

Lamping '632 therefore do not indicate "an angular difference between an incoming link to the parent and an outgoing link from the parent to the element" as called for in claim 19.

Accordingly, claim 19 should be patentable in its own right.

b. **Claim 20** depends from claim 19 and adds the further limitation that the layout data include *only* the position displacement data and the angle displacement data. Again, Lamping '632 does not teach this limitation, and the Examiner has not made any attempt to identify it.

c. **Claim 21** calls for the step of obtaining nearby relationship data to include a step of obtaining a count of grandchildren for each of a set of children of the parent. Nothing in Lamping '632 teaches this feature and the Examiner has not provided any argument why it does. The grounds for rejection in the August 12, 2002 Office Action are:

> Re claims 6-7 and 21-23, Lamping discloses the radii and angles for the set of children to obtain a position displacement and an angle displacement between the parent and the element (col. 23 and 24; fig. 13).

(August 12, 2002 Office Action, p. 5, lines 1-3.)

Again, this does not make even a prima facie case that claim 21 is unpatentable, since it mentions nothing about a "count of grandchildren". And the cited section of Lamping '632 describes a process of *mapping* the information about the node-link structure *from* layout space to two-dimensional display space, which is *not a space with negative curvature*. The cited sections therefore are not a method of laying out a node-link structure in a space with negative curvature.

Accordingly, claim 21 should be patentable in its own right.

d. **Claim 22** depends from claim 21 and adds limitations that the counts of grandchildren used in obtaining the layout data, be used to obtain radii and angles, and that

position displacement and angle displacement between the parent and the element be obtained using the radii and angles. Again, Lamping '632 neither teaches nor suggests this feature and the Examiner has not provided any argument why it does.

e. **Claim 23** depends from claim 22 and adds a limitation calling for a comparison to be made between the obtained angle displacement and the previous angle displacement to determine whether to lay out children of the element. Again, Lamping '632 nowhere teaches this element, and the parts of Lamping '632 cited by the Examiner (col. 23 and 24, fig. 13) do not describe a method of laying out a node-link structure in a space with negative curvature.

f. **Claim 24** depends from claim 17 and adds a limitation calling for the nearby node-link relationships, based on which the element's position relative to a parent in the space with negative curvature is to be obtained, include *only* relationships among the parent and the parent's children and grandchildren. Lamping '632 does not teach or suggest this feature.

In the August 12, 2002 Office Action, at p. 5, lines 4-6, the Examiner pointed to Lamping '632, col. 25, lines 24-50 as teaching this feature. However, again, that part of Lamping '632 concerns the mapping step of Lamping '632, not the method of laying out a node-link structure in a space with negative curvature.

Accordingly, it is respectfully submitted that each of the dependent claims 18-25 should be patentable since the Examiner has not made a prima facie case of unpatentability.

3. Independent Claim 29, as Representative of Independent Claims 29 and 42-44 and Dependent Claims 30 and 37-41

As mentioned, whereas independent claim 17 focuses in primarily on the *localness* of the layout methods, representative independent claim 29 focuses in primarily on the *relativeness* of the positions stored for the elements in the layout space. Claim 29 calls for:

> 29. A method of laying out a plurality of elements of a node-link structure in a space with negative curvature, the method comprising:
>
> obtaining nearby relationship data for <u>each</u> element in the plurality, the nearby relationship data indicating information about nearby node-link relationships;
>
> based on the nearby relationship data for each element in the plurality, calculating element's position in the space with negative curvature; and
>
> storing the positions for each element in the plurality in a data structure such that after the positions for <u>all</u> elements in the plurality have been calculated, the position of <u>each</u> element in the plurality is stored in the data structure <u>only relative to an element</u> of the node-link structure <u>other than a root</u> element of the node-link structure. (emphasis added).

Thus claim 29, like claim 17, at a minimum calls for a step of calculating an element's position in a <u>space with negative curvature</u>.

The Examiner rejected claim 29 initially in the January 13, 2004 Office Action, at p. 4, line 18 - p. 5, line 10, citing Lamping '632, col. 23, line 56 to col. 24, line 65; col. 16, lines 25-62; and Figs. 13-21. This citation was repeated in each subsequent Office Action.

But as with the rejection of claim 17, these parts of Lamping '632 have to do with a method for mapping from *layout* space to a circular *display* region, which is flat, and is not a space with negative curvature.

Specifically, the Examiner cites Lamping's Fig. 13 and the description at col. 23, line 56 - col. 24, line 65. But Fig. 13 begins *after* the step in which the transformation is made from node-link data to layout data (i.e., *after* step 304 of Lamping's Fig. 8). See the very beginning of Fig. 13: "FROM BOX 304, ..."

In Fig. 13, the procedure described has to do with mapping *from* the layout data to the *flat unit disk* for eventual display. See step 500 at the beginning of Fig. 13: "Receive call for *mapping* handle of root node." See also the beginning of the description at col. 23, lines 38-40: "Fig. 13 shows acts in *using* layout data *from* box 304 in Fig. 8 to present a representation of a node-link structure, as in boxes 306, 326, and 336 in Fig. 8." Boxes 306, 326, and 336 in Fig. 8 are all steps for *mapping* and *presenting* representations of the structure on a display or printout.

Thus as with independent claim 17, the Examiner has cited nothing in Lamping '632 that teaches *anything* about calculating positions in a space with negative curvature, only excerpts about obtaining *mapped* positions on the unit disk, which is *flat*, not a space with negative curvature.

In a telephone interview on July 2, 2003, Appellants thought that the Examiner had changed her position and was then of the view that the *layout* step of Lamping '632, and more specifically Lamping '250 as incorporated into Lamping '632, taught Appellants' previously claimed step of obtaining layout data indicating an element's position relative to a parent in the space with negative curvature. In their next Response, Response D filed October 21, 2003, therefore, at pp. 15-19, Appellants provided a detailed analysis of the layout method taught in

Lamping '250 and Lamping '632 and showed why that method would not satisfy the limitations of Appellants' claim 29. The Examiner appears not to have repeated this argument in the subsequent Office Actions, but should it be repeated in the Examiner's Answer, the Board is respectfully referred to the above section of Appellants' Response D.

Accordingly, Appellants respectfully submit that the Examiner has yet to make a *prima facie* case of unpatentability of claim 29.

4. Dependent Claims 31-36

Claims 31-36 all depend ultimately from independent claim 29. These claims should all be allowable because of their dependency from independent claim 29. These claims also each add their own limitations which, it is submitted, render them patentable in their own right.

a. **Claim 31** adds a limitation that the position of each particular element as represented in the data structure after the positions for all elements in the plurality have been calculated, include:

> <u>position displacement data</u> indicating a distance between the particular element and a parent of the particular element,

and

> <u>angle displacement data</u> indicating an <u>angular difference</u> between an incoming link to the parent of the particular element and an outgoing link from the parent to the particular element. (emphasis added)

Nowhere does Lamping '632 teach such features. At a minimum, the position in Lamping '632 specifies *angle* information only relative to a predetermined and fixed "zero direction". Lamping '250, col. 23, lines 64-65, referenced in Lamping '632. He certainly neither teaches nor suggests representing layout data using "angle displacement data indicating an angular difference

between an incoming link to the parent of the particular element and an outgoing link from the parent to the particular element" as called for in Appellants' claim 31. The angles of incoming and outgoing links for parent nodes in layout space are not even mentioned in Lamping '632.

In the outstanding office action, the Examiner cites Lamping col. 23-24 and Fig. 13 as disclosing "the radii and angles for the set of children to obtain a position displacement and an angle displacement between the parent and the element." (October 6, 2004 Office Action, p. 10, lines 3-5.) But again, the cited text of Lamping concerns node positions in the flat display space, not in the space with negative curvature as required by parent claim 29.

b. **Claim 32** depends from claim 31 and adds the further limitation that the position as represented in the data structure include *only* the position displacement data and the angle displacement data. Again, Lamping '632 does not teach this limitation.

In the outstanding office action, the Examiner again cites Lamping col. 23-24 and Fig. 13 as teaching this limitation. (October 6, 2004 Office Action, p. 10, lines 3-5.) But again, the cited text of Lamping concerns node positions in the flat display space, not in the space with negative curvature as required by parent claim 29.

c. **Claim 33** calls for the step of obtaining nearby relationship data to include a step of obtaining a count of grandchildren for each of a set of children of the parent. Nothing in Lamping '632 teaches this feature. Nor does the Examiner make any attempt to identify it, except to point to col. 25, lines 24-50 and Fig. 13 (October 6, 2004 Office Action, p. 10, lines 6-8), which again concerns node positions in the flat display space, not in the space with negative curvature as required by parent claim 29.

d. **Claim 34** depends from claim 33 and adds limitations that the counts of grandchildren be used to obtain radii and angles, and that position displacement and angle

displacement between the parent and the element be obtained using the radii and angles. Again, Lamping '632 neither teaches nor suggests this feature. Nor does the Examiner in the office action make any attempt to identify it, except to point again to Lamping, col. 25, lines 24-50 and Fig. 13 (October 6, 2004 Office Action, p. 10, lines 6-8), which again concerns node positions in the flat display space, not in the space with negative curvature as required by parent claim 29.

e. **Claim 35** depends from claim 34 and adds a limitation calling for a comparison to be made between the obtained angle displacement and the previous angle displacement to determine whether to lay out children of the element. Again, Lamping '632 nowhere teaches this element, and the part of Lamping '632 cited by the Examiner (col. 23 and 24; Fig. 13; cited in October 6, 2004 Office Action, p. 10, lines 3-5) is not a method of laying out a node-link structure in a space with negative curvature.

f. **Claim 36** depends from claim 30 and adds a limitation calling for the nearby node-link relationships, based on which the element's position relative to a parent in the space with negative curvature is to be obtained, include only relationships among the parent and the parent's children and grandchildren. The nearby node-link relationships therefore do not include any ancestors of the element's parent element. Not only is this feature not taught in Lamping '632, but it is contrary to Lamping '632, in which the node-link structure is taught as being laid out beginning at the root node. Nor do the parts of Lamping '632 identified by the Examiner (col. 25, lines 24-50; Fig. 13, cited in October 6, 2004 Office Action, p. 10, lines 6-8) teach the claimed features.

Accordingly, it is respectfully submitted that each of the dependent claims 31-36 should be patentable since the Examiner has not made a prima facie case of unpatentability.

5. Conclusion of Argument

Appellants first invented a novel and effective way of helping users to visualize very large node-link structures, and patented their methods in the Lamping '632 and Lamping '250 patents, among others. Roughly described, the methods involve a two-step process: laying out the node-link structure in a layout space with negative curvature, and then mapping the elements from layout space to a flat space for displaying on the surface of a flat monitor.

They later saw a problem with the initial implementation of their method, and discovered a substantial improvement in the first of these two steps, the layout step, that could be made in order to overcome it. They filed the present patent application to claim the new improvement.

The Examiner, however, is refusing to allow Appellants' claims because of an erroneous belief that the improvement is not only obvious from, but anticipated by, the inventors' own earlier Lamping '632 patent.

And for evidence the Examiner cites sections of Lamping '632 that describe the *second* step of Appellants' method, the *mapping* step, refusing to acknowledge that the claims call specifically for a method to take place in the *first* step of the method, the *layout* step. The claims do not require the mapping step of Lamping '632 to follow, but they do call unambiguously for determining the position of an element of the node-link structure in a space with negative curvature, which the mapping step of Lamping '632 does not do.

Anticipation requires that every element of the claim be taught in a single cited reference. The Examiner has so far inexplicably failed to do this.

IX. CONCLUSION

In view of the foregoing, Appellants ask that this honorable Board reverse the Examiner's rejections of the claims. In addition, it is submitted that all claims that are the subject of this examination are now allowable, and a notice of intent to issue a patent is respectfully requested.

The Commissioner is hereby authorized to charge any fee determined to be due in connection with this communication, or credit any overpayment, to our Deposit Account No. 50-0869 (File No. INXT 1002-1).

Respectfully submitted,

Dated: 4 November 2005

Warren S. Wolfeld

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X. CLAIMS APPENDIX

1-16. (Canceled)

17. (previously presented) A method of laying out a node-link structure in a space with negative curvature; the method comprising:

obtaining nearby relationship data for a subject element in the structure, the nearby relationship data indicating information about nearby node-link relationships, the nearby relationship data excluding relationships with at least one element of the node-link structure; and

based on only the nearby relationship data, and not on the position of any other element in the structure, obtaining layout data identifying the subject element's position in the space with negative curvature.

18. (original) A method as in claim 17 in which the space with negative curvature is a hyperbolic plane.

19. (previously presented) A method as in claim 17 wherein the subject element has a parent element in the node-link structure, and in which the subject element and the parent are nodes and in which the layout data include position displacement data indicating a distance between the parent's position and the subject element's position and angle displacement data indicating an angular difference between an incoming link to the parent and an outgoing link from the parent to the subject element.

20. (original) A method as in claim 19 in which the layout data include only the position displacement data and the angle displacement data.

21. (previously presented) A method as in claim 17 wherein the subject element has a parent element in the node-link structure, and in which the act of obtaining the nearby relationship data comprises:

for each of a set of children of the parent, obtaining a count of grandchildren; the subject element being one of the set of children.

22. (original) A method as in claim 21 in which the act of obtaining layout data comprising:

using the counts of grandchildren to obtain, for each of the set of children, a radius and an angle; and

using the radii and angles for the set of children to obtain a position displacement and an angle displacement between the parent and the subject element.

23. (original) A method as in claim 22 in which the subject element has a previous angle displacement; the method further comprising comparing the obtained angle displacement with the previous angle displacement to determine whether to lay out children of the subject element.

24. (previously presented) A method as in claim 17 wherein the subject element has a parent element in the node-link structure, and in which the nearby node-link relationships include only relationships among the parent and the parent's children and grandchildren.

25. (original) A method according to claim 17, wherein said step of obtaining layout data comprises the step of calculating said layout data.

26. (previously presented) A system comprising:

a processor for laying out a node-link structure in a space with negative curvature; the processor, in laying out the node-link structure:

obtaining nearby relationship data for a subject element in the structure, the nearby relationship data indicating information about nearby node-link relationships, the nearby relationship data excluding relationships with at least one element of the node-link structure; and

based on only the nearby relationship data, and not on the position of any other element in the structure, obtaining layout data identifying the subject element's position in the space with negative curvature.

27. (previously presented) An article of manufacture for use in a system that includes: a storage medium access device; and

a processor connected for receiving data accessed on a storage medium by the storage medium access device; the article of manufacture comprising:

a storage medium; and

instruction data stored by the storage medium; the instruction data indicating instructions the processor can execute; the processor, in executing the instructions, laying out a node-link structure in a space with negative curvature; the processor, in laying out the node-link structure:

obtaining nearby relationship data for a subject element in the structure, the nearby relationship data indicating information about nearby node-link relationships, the nearby relationship data excluding relationships with at least one element of the node-link structure; and

based on only the nearby relationship data, and not on the position of any other element in the structure, obtaining layout data identifying the subject element's position in the space with negative curvature.

28. (previously presented) A method of transferring data between first and second machines over a network, the second machine including memory and a processor connected for accessing the memory; the memory being for storing instruction data; the method comprising:

establishing a connection between the first and second machines over the network; and

operating the first and second machines to transfer instruction data from the first machine to the memory of the second machine; the instruction data indicating instructions the processor can execute; the processor, in executing the instructions, laying out a node-link structure in a space with negative curvature; the processor, in laying out the node-link structure:

obtaining nearby relationship data for a subject element in the structure, the nearby relationship data indicating information about nearby node-link relationships, the nearby relationship data excluding relationships with at least one element of the node-link structure; and

based on only the nearby relationship data, and not on the position of any other element in the structure, obtaining layout data identifying the subject element's position in the space with negative curvature.

29. (previously presented) A method of laying out a plurality of elements of a node-link structure in a space with negative curvature, the method comprising:

obtaining nearby relationship data for each element in the plurality, the nearby relationship data indicating information about nearby node-link relationships;

based on the nearby relationship data for each element in the plurality, calculating element's position in the space with negative curvature; and

storing the positions for each element in the plurality in a data structure such that after the positions for all elements in the plurality have been calculated, the position of each element in the plurality is stored in the data structure only relative to an element of the node-link structure other than a root element of the node-link structure.

30. (previously presented) A method according to claim 29, wherein said step of storing comprises the step of storing the positions for each element in the plurality in a data

structure such that after the positions for all elements in the plurality have been calculated, the position of each element in the plurality is stored in the data structure only relative to a parent of the element.

31. (previously presented) A method as in claim 30, in which the elements in the plurality of elements are nodes, and in which the parents are nodes, and in which the position of each particular element in the plurality as represented in the data structure after the positions for all elements in the plurality have been calculated, includes position displacement data indicating a distance between the particular element and a parent of the particular element, and angle displacement data indicating an angular difference between an incoming link to the parent of the particular element and an outgoing link from the parent to the particular element.

32. (previously presented) A method as in claim 31, in which the position of each particular element in the plurality as represented in the data structure after the positions for all elements in the plurality have been calculated, includes only the position displacement data and the angle displacement data.

33. (previously presented) A method as in claim 30, in which the step of obtaining the nearby relationship data comprises, for a particular one of the elements in the plurality:

for each of a set of children of the parent of the particular element, obtaining a count of grandchildren, the particular element being one of the set of children.

34. (previously presented) A method as in claim 33, in which the step of obtaining layout data comprises, for the particular element:

using the counts of grandchildren to obtain, for each of the set of children, a radius and an angle; and

using the radii and angles for the set of children to obtain a position displacement and an angle displacement between the parent and the particular element.

35. (previously presented) A method as in claim 34, in which the particular element has a previous angle displacement, the method further comprising the step of comparing the obtained angle displacement with the previous angle displacement to determine whether to lay out children of the particular element.

36. (previously presented) A method as in claim 30, in which the nearby node-link relationships include only relationships among the parent and the parent's children and grandchildren.

37. (previously presented) A method as in claim 29, in which the method is performed in each of a series of iterations, each iteration comprising the steps of:

identifying elements to be laid out in the iteration;

performing the steps of obtaining and calculating for each of the identified elements; and

performing the step of storing for the identified elements.

38. (previously presented) A method as in claim 37, in which the series of iterations is performed in response to an event requesting an insertion or deletion, the identified elements including elements affected by the insertion or deletion.

39. (previously presented) A method as in claim 38, further comprising, before the series of iterations, the step of obtaining a weight for each iteration,

each iteration comprising using the weight in performing the step of calculating each element's position in the space with negative curvature.

40. (previously presented) A method as in claim 37, in which the identified elements include elements added to the structure during a preceding iteration.

41. (previously presented) A method as in claim 29, in which the space with negative curvature is a hyperbolic plane.

42. (previously presented) A system comprising:

a processor for laying out a plurality of elements of a node-link structure in a space with negative curvature, the processor, in laying out the node-link structure:

obtaining nearby relationship data for each element in the plurality, the nearby relationship data indicating information about nearby node-link relationships;

based on the nearby relationship data for each element in the plurality, calculating element's position in the space with negative curvature; and

storing the positions for each element in the plurality in a data structure such that after the positions for all elements in the plurality have been calculated, the position of each element in the plurality is stored in the data structure only relative to an element of the node-link structure other than a root element of the node-link structure.

43. (previously presented) An article of manufacture for use in a system that includes a storage medium access device and a processor connected for receiving data accessed on a storage medium by the storage medium access device, the article of manufacture comprising:

a storage medium; and

instruction data stored by the storage medium, the instruction data indicating instructions the processor can execute, the processor, in executing the instructions, laying out a plurality of elements of a node-link structure in a space with negative curvature, the processor, in laying out the plurality of elements:

obtaining nearby relationship data for each element in the plurality, the nearby relationship data indicating information about nearby node-link relationships;

based on the nearby relationship data for each element in the plurality, calculating element's position in the space with negative curvature; and

storing the positions for each element in the plurality in a data structure such that after the positions for all elements in the plurality have been calculated, the position of each element in the plurality is stored in the data structure only relative to an element of the node-link structure other than a root element of the node-link structure.

44. (previously presented) A method of transferring data between first and second machines over a network, the second machine including memory and a processor connected for accessing the memory, the memory being for storing instruction data, the method comprising the steps of:

establishing a connection between the first and second machines over the network; and

operating the first and second machines to transfer instruction data from the first machine to the memory of the second machine, the instruction data indicating instructions the processor can execute, the processor, in executing the instructions, laying out a plurality of elements of a node-link structure in a space with negative curvature, the processor, in laying out the plurality of elements:

obtaining nearby relationship data for each element in the plurality, the nearby relationship data indicating information about nearby node-link relationships;

based on the nearby relationship data for each element in the plurality, calculating element's position in the space with negative curvature; and

storing the positions for each element in the plurality in a data structure such that after the positions for all elements in the plurality have been calculated, the position of each element in the

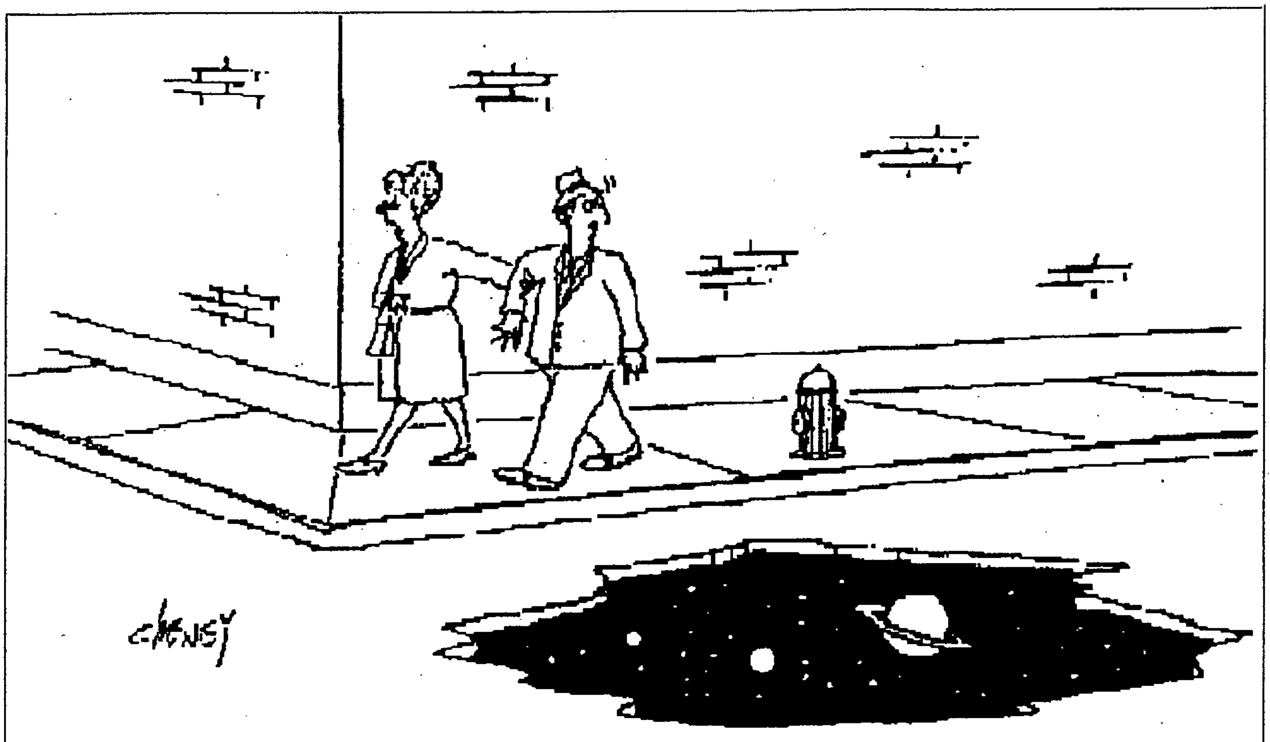
plurality is stored in the data structure only relative to an element of the node-link structure other than a root element of the node-link structure.

XI. EVIDENCE APPENDIX

REFERENCE	WHERE ENTERED IN RECORD	Exhibit
"Space Properties", pp. 1-6 (2002), available at http://scholar.uwinnipeg.ca/courses/38/4500.6-001/Cosmology/Properties_of_Space.pdf, visited 6/4/2004	Copy attached to Response E, filed July 13, 2004	1
Anderson, James W., "Hyperbolic Geometry," Springer-Verlag London Limited, Great Britain, 1999, title page, copyright page, and pp. 1-7	IDS submitted April 6, 2005	2
U.S. Patent No. 5,619,632	IDS submitted November 12. 1999	3
U.S. Patent No. 5,590,250	IDS submitted November 12, 1999	4

Space Properties

Up Next



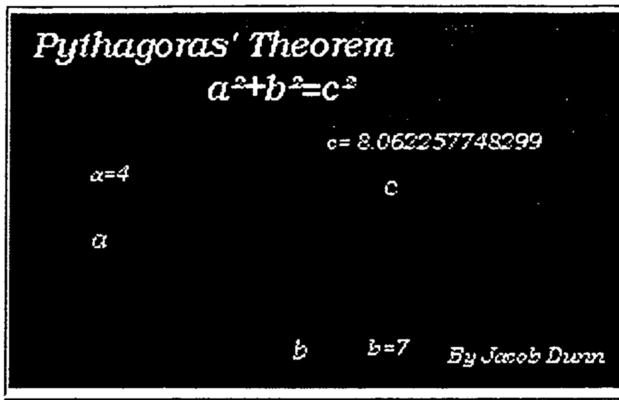
Oh, for heavens sake, Norman! You act as if you have never seen a hole in the time-space continuum before.

Three properties of space will be discussed:

Geometry, Topology and Dimensionality

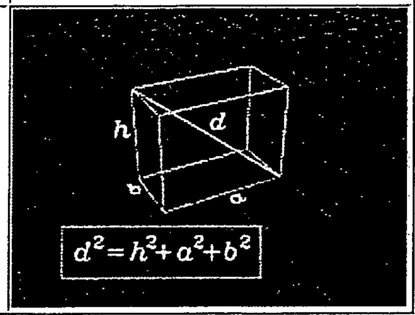
Euclidean and Non-Euclidean Geometry

In one area of human inquiry there had long existed a quiet confidence in our ability to fathom something of the ultimate truth about the universe. People thought that if this success was possible in one area of inquiry then perhaps it was true in others. The source of this confidence was the age-old study of geometry that Euclid and the ancient Greeks had placed upon a firm logical foundation.



Euclidean geometry is a geometry where the Pythagoras Theorem for triangles holds. The theorem gives the distance-squared between two points (c^2 in the diagram) as the sum of the squares of the other two sides (a^2 and b^2). Any space where this Euclidean distance function holds is said to be spatially flat.

The distance function can be generalized to any number of spatial dimensions using exactly the pattern that was used above. In the 3-D case one merely has to add one more independent variable. In the diagram to the right the symbol h represents this new variable. The variable h is what we normally refer to as the height.

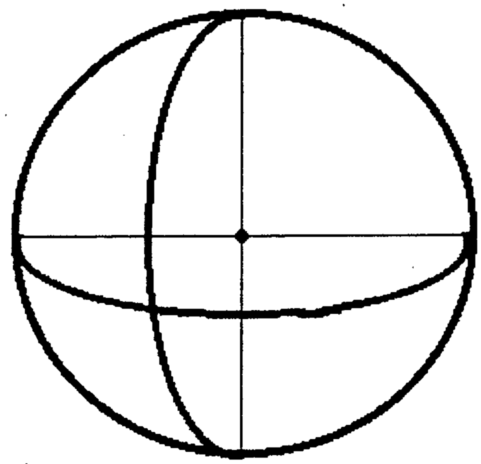


Euclid's geometry had done more than help architects and cartographers. It had established a style of reasoning, wherein truths were deduced by the application of definite rules of reasoning from a collection of self-evident axioms. Theology and philosophy copied this 'axiomatic method', and most forms of philosophical argument followed its general pattern. In extreme cases, as in the work of the Dutch philosopher Spinoza, philosophical propositions were even laid out like the definitions, axioms, theorems, and proofs in Euclid's works.

Euclidean geometry was believed to be a description of how the world was, it was not an approximation, it was not a human construct; it was the *absolute truth.* Theologians pointed to it as a rationale for why absolute truth inquiries were legitimate.

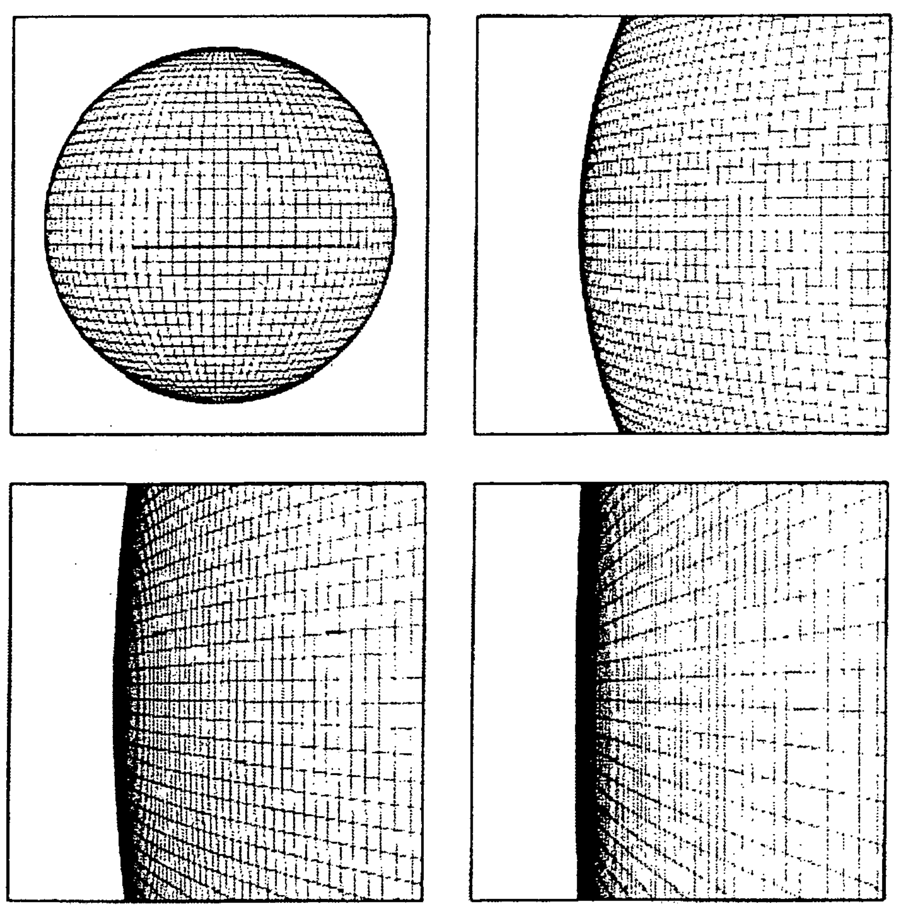
In the late 1700's Immanuel Kant , the great German philosopher declared that Euclid's geometry was true independent of experience.

In the first quarter of the 1800's things started to change. The confidence in Euclid's geometry was starting to be undermined. The famous mathematician Carl Gauss asserted that geometry was dictated by experiment (experiment ---> axioms). There were other possible geometries describing the logical interrelationships between points and lines on curved surfaces. These geometries could have practical applications. Long distance travel on the curved surface of the Earth required non-Euclidean geometry.



Why were non-Euclidean geometries so unfamiliar? Why are they still unfamiliar? The reason is that most people are restricted to small portions of the Earth's surface and usually the curvature of the Earth is negligibly small in these regions. A bricklayer or a carpenter must use Euclidean geometry, but an ocean going yachtsman cannot.

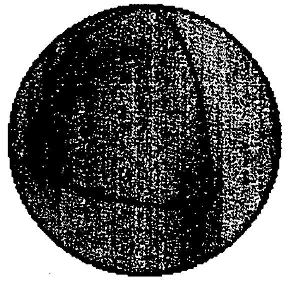
GLOBAL: On the large scale, the curvature of the structure is evident.



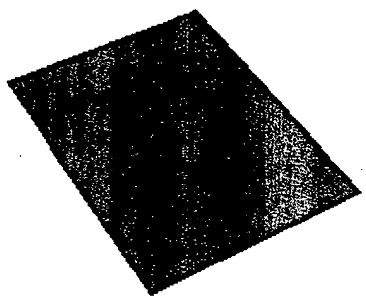
LOCAL: On the small scale, the curvature of the structure is not easily seen.

The curvature of space can be classified into three types: spherical (positive curvature), flat (zero curvature), and hyperbolic (negative curvature).

In spherical curvature the angles inside triangles add up to be more than 180 degrees.



In flat space the angles of a triangle add up to be 180 degrees exactly.

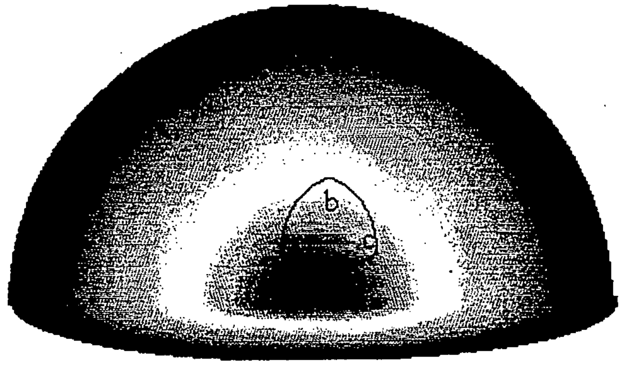


In a hyperbolic space the inside angles of a triangle add up to be less than 180 degrees.

These ideas are summarized in the following diagram.

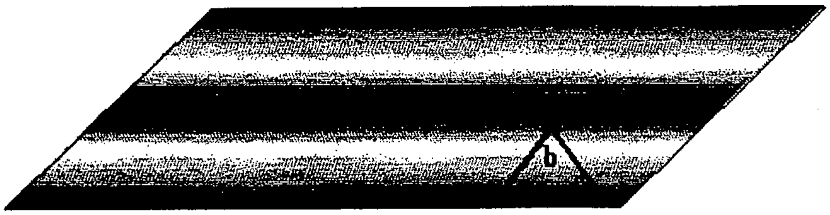
Spherical space

a + b + c > 180
curvature = positive



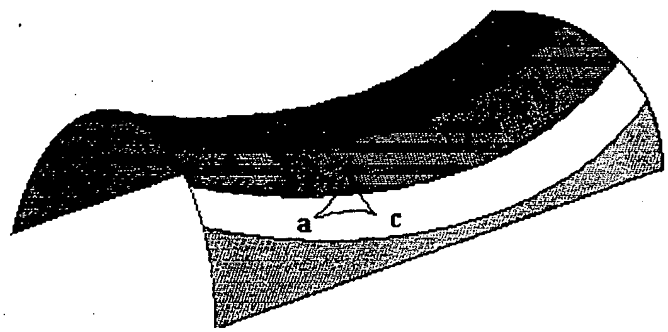
Flat Space

a + b + c = 180
curvature = 0

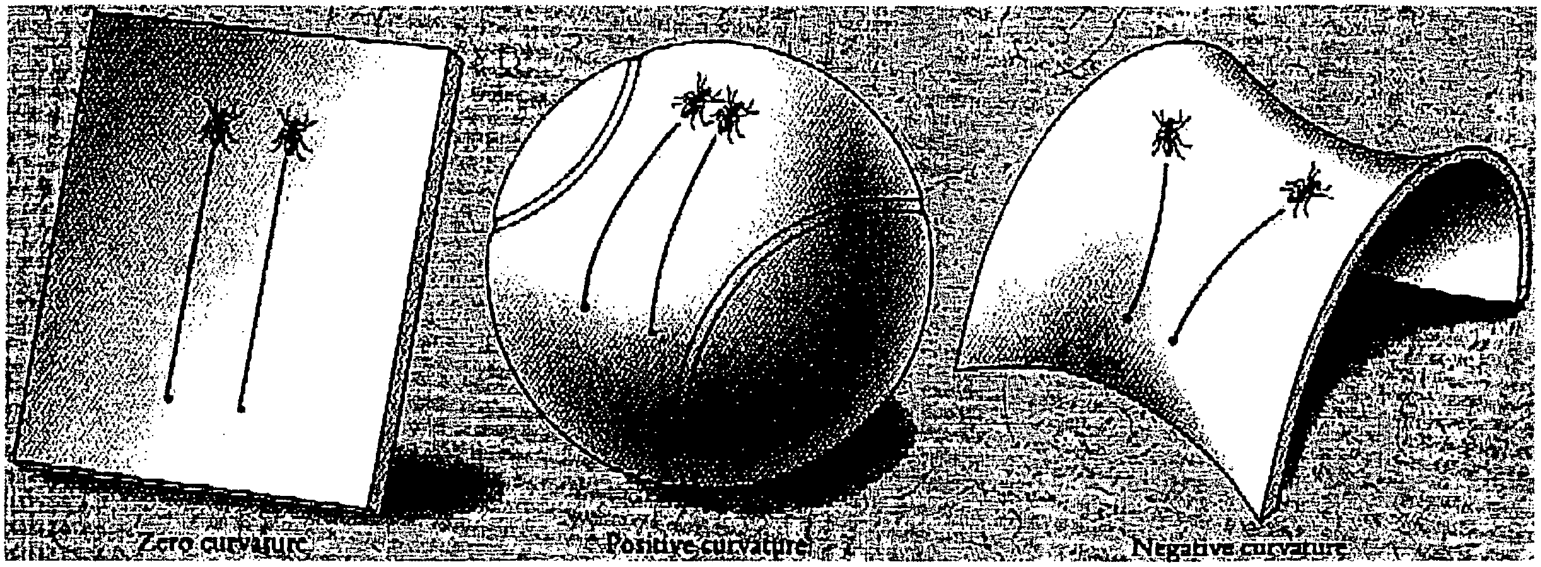


Hyperbolic space

a + b + c < 180
curvature = negative



In Euclid's geometry all surfaces are flat and parallel lines always stay the same distance apart, never meeting and never diverging. However, in curved non-Euclidean geometries, lines that start off parallel eventually cross each other in the positive curvature case, while these same lines diverge from each other in the negative curvature case. This is illustrated below in the case of two ants traveling along the three types of curved surface.

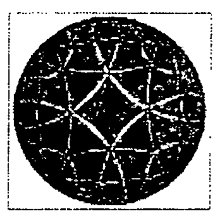


Geometry of space is of great importance to cosmology since Einstein's General Theory of Relativity, which will be discussed later, relies entirely on the idea that the the geometry of space at any location in the universe is directly related to the strength of the gravitational field at that location. The stronger the gravitational field is, then the stronger will be the matching curvature. In a cosmological context, the three curvature types are

The positive curvature universe corresponds to a universe that will expand to a certain separation between galaxies and then contract back to zero space. This is called a closed universe.

The zero curvature universe corresponds to a universe that will expand forever, slowing down as it does so. This is called a spatially flat universe.

The negative curvature universe corresponds to a universe that will expand forever. This is called an open universe. For a Escher print based on the concept of negative curvature click on the following small picture.



Topology

Topology is the branch of mathematics concerned with the ramifications of continuity. Topologist emphasize the properties of shapes that remain unchanged no matter how much the shapes are bent twisted or otherwise manipulated.

Such transformations of ideally elastic objects are subject only to the condition that, for surfaces, nearby points remain close together in the transforming process. This condition effectively outlaws transformations that involve cutting and gluing. For instance a doughnut and a coffee cup are topologically equivalent. One can be transformed continuously into the other. The hole in the doughnut will be preserved as the hole in the handle of the coffee cup.

James W. Anderson

Hyperbolic Geometry

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$|\langle \chi, \gamma \rangle| \leq \|\chi\| \|\gamma\|$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$\int_a^b f(t)\,dt = F(b) - F(a)$$

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Hyperbolic Geometry

With 20 Figures

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1
The Basic Spaces

In this first chapter, we set the stage for what is to come. Namely, we define the *upper half-plane model* $\mathbb{H}$ of the hyperbolic plane, which is where most of the action of this book takes place. We define *hyperbolic lines* and talk a bit about *parallelism*. In order to aid our construction of a reasonable group of transformations of $\mathbb{H}$, we expand our horizons to consider the *Riemann sphere* $\overline{\mathbb{C}}$, and close the chapter by considering how $\mathbb{H}$ sits as a subset of $\overline{\mathbb{C}}$.

1.1 A Model for the Hyperbolic Plane

We begin our investigation by describing a model of the hyperbolic plane. By a *model*, we mean a choice of an underlying space and a choice of how to represent basic geometric objects, such as points and lines, in this underlying space. As we shall see over the course of the book, there are a large number of possible models for the hyperbolic plane. In order to give as concrete a description of its geometry as possible, we begin by working in a single specific model.

The model of the hyperbolic plane we work in is the *upper half-plane* model. The underlying space of this model is the upper half-plane $\mathbb{H}$ in the complex plane $\mathbb{C}$, defined to be

$$\mathbb{H} = \{z \in \mathbb{C} : \mathrm{Im}(z) > 0\}.$$

We use the usual notion of point that $\mathbb{H}$ inherits from $\mathbb{C}$. We also use the usual notion of angle that $\mathbb{H}$ inherits from $\mathbb{C}$; that is, the angle between two curves in $\mathbb{H}$ is defined to be the angle between the curves when they are considered to be curves in $\mathbb{C}$, which in turn is defined to be the angle between their tangent lines.

As we will define hyperbolic lines in $\mathbb{H}$ in terms of Euclidean lines and Euclidean circles in $\mathbb{C}$, we begin with a couple of calculations in $\mathbb{C}$.

Exercise 1.1

Express the equations of the Euclidean line $ax + by + c = 0$ and the Euclidean circle $(x-h)^2+(y-k)^2 = r^2$ in terms of the complex coordinate $z = x + yi$ in $\mathbb{C}$.

Exercise 1.2

Let $\mathbb{S}^1 = \{z \in \mathbb{C} \mid |z| = 1\}$ be the unit circle in $\mathbb{C}$. Let A be a Euclidean circle in $\mathbb{C}$ with Euclidean centre $re^{i\theta}$, $r > 1$, and Euclidean radius $s > 0$. Show that A is perpendicular to $\mathbb{S}^1$ if and only if $s = \sqrt{r^2 - 1}$.

We are now ready to define a *hyperbolic line* in $\mathbb{H}$.

Definition 1.1

There are two seemingly different types of *hyperbolic line*, both defined in terms of Euclidean objects in $\mathbb{C}$. One is the intersection of $\mathbb{H}$ with a Euclidean line in $\mathbb{C}$ perpendicular to the real axis $\mathbb{R}$ in $\mathbb{C}$. The other is the intersection of $\mathbb{H}$ with a Euclidean circle centred on the real axis $\mathbb{R}$.

Some examples of hyperbolic lines in $\mathbb{H}$ are shown in Fig. 1.1.

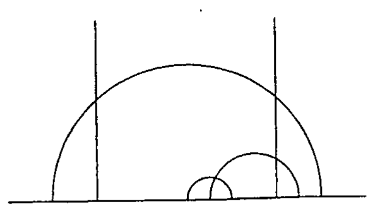

Figure 1.1: Hyperbolic lines in $\mathbb{H}$

We will see in Section 1.2 a way of unifying these two different types of hyperbolic line. For the moment, though, we content outselves with an exploration of some of the basic properties of hyperbolic geometry with this definition of hyperbolic line.

Working in analogy with what we know from Euclidean geometry, there is one property that hyperbolic lines in $\mathbb{H}$ should have, namely that there should always exist one and only one hyperbolic line between any pair of distinct points of $\mathbb{H}$. That this property holds in $\mathbb{H}$ with hyperbolic lines as defined above is a fairly straightforward calculation.

Proposition 1.2

For each pair p and q of distinct points in $\mathbb{H}$, there exists a unique hyperbolic line ℓ in $\mathbb{H}$ passing through p and q.

There are two cases to consider. Suppose first that $\mathrm{Re}(p) = \mathrm{Re}(q)$. Then, the Euclidean line L given by the equation $L = \{z \in \mathbb{C} \,|\, \mathrm{Re}(z) = \mathrm{Re}(p)\}$ is perpendicular to the real axis and passes through both p and q. So, the hyperbolic line $\ell = \mathbb{H} \cap L$ is the desired hyperbolic line through p and q.

Suppose now that $\mathrm{Re}(p) \neq \mathrm{Re}(q)$. Since the Euclidean line through p and q is no longer perpendicular to $\mathbb{R}$, we need to construct a Euclidean circle centred on the real axis $\mathbb{R}$ that passes though p and q.

Let L_{pq} be the Euclidean line segment joining p and q and let K be the perpendicular bisector of L_{pq}. Then, every Euclidean circle that passes through p and q has its centre on K. Since p and q have non-equal real parts, the Euclidean line K is not parallel to $\mathbb{R}$, and so K and $\mathbb{R}$ intersect at a unique point c.

Let A be the Euclidean circle centred at this point of intersection c with radius $|c-p|$, so that A passes through p. Since c lies on K, we have that $|c-p| = |c-q|$, and so A passes through q as well. The intersection $\ell = \mathbb{H} \cap A$ is then the desired hyperbolic line passing through p and q.

The uniqueness of the hyperbolic line passing through p and q comes from the uniqueness of the Euclidean lines and Euclidean circles used in its construction. This completes the proof of Proposition 1.2.

We note here that the argument used to prove Proposition 1.2 actually contains more information. For any pair of distinct points p and q in $\mathbb{C}$ with non-equal real parts, there exists a unique Euclidean circle centred on $\mathbb{R}$ passing through p and q. The crucial point is that the centre of any Euclidean circle passing through p and q lies on the perpendicular bisector K of the Euclidean line segment L_{pq} joining p and q, and K is not parallel to $\mathbb{R}$.

Since we have chosen the underlying space $\mathbb{H}$ for this model of the hyperbolic plane to be contained in $\mathbb{C}$, and since we have chosen to define hyperbolic lines in $\mathbb{H}$ in terms of Euclidean lines and Euclidean circles in $\mathbb{C}$, we are able to use whatever facts about Euclidean lines and Euclidean circles we know to analyze the behaviour of hyperbolic lines. We have in effect given ourselves familiar coordinates on $\mathbb{H}$ to work with.

For instance, if ℓ is the hyperbolic line in $\mathbb{H}$ passing through p and q, we are able to express ℓ explicitly in terms of p and q. When p and q have equal real parts, we have already seen that $\ell = \mathbb{H} \cap L$, where L is the Euclidean line $L = \{z \in \mathbb{C} \mid \mathrm{Re}(z) = \mathrm{Re}(p)\}$. The expression of ℓ in terms of p and q in the case that $\mathrm{Re}(p) \neq \mathrm{Re}(q)$ is left as an exercise.

Exercise 1.3

Let p and q be distinct points in $\mathbb{C}$ with non-equal real parts and let A be the Euclidean circle centred on $\mathbb{R}$ and passing through p and q. Express the Euclidean centre c and the Euclidean radius r of A in terms of $\mathrm{Re}(p)$, $\mathrm{Im}(p)$, $\mathrm{Re}(q)$, and $\mathrm{Im}(q)$.

A legitimate question to raise at this point is whether hyperbolic geometry in $\mathbb{H}$, with this definition of hyperbolic line, is actually different from the usual Euclidean geometry in $\mathbb{C}$ we are accustomed to. The answer to this question is an emphatic Yes, hyperbolic geometry in $\mathbb{H}$ behaves very differently from Euclidean geometry in $\mathbb{C}$.

One way to see this difference is to consider the behaviour of parallel lines. Recall that Euclidean lines in $\mathbb{C}$ are parallel if and only if they are disjoint, and we adopt this definition in the hyperbolic plane as well.

Definition 1.3

Two hyperbolic lines are *parallel* if they are disjoint.

In Euclidean geometry, parallel lines exist, and in fact, if L is a Euclidean line and if a is a point in $\mathbb{C}$ not on L, then there exists one and only one line K through a that is parallel to L.

In fact, in Euclidean geometry parallel lines are also equidistant, that is, if L and K are parallel Euclidean lines and if a and b are points on L, then the Euclidean distance from a to K is equal to the Euclidean distance from b to K.

In hyperbolic geometry, parallelism behaves much differently. Though we do not yet have a means of measuring hyperbolic distance, we can consider parallel hyperbolic lines qualitatively.

Theorem 1.4

Let ℓ be a hyperbolic line in $\mathbb{H}$ and let p be a point in $\mathbb{H}$ not on ℓ. Then, there exist infinitely many different hyperbolic lines through p that are parallel to ℓ.

As in the proof of Proposition 1.2, there are two cases to consider. First, suppose that ℓ is contained in a Euclidean line L. Since p is not on L, there exists a Euclidean line K through p that is parallel to L. Since L is perpendicular to $\mathbb{R}$, we have that K is perpendicular to $\mathbb{R}$ as well. So, one hyperbolic line in $\mathbb{H}$ through p and parallel to ℓ is the intersection $\mathbb{H} \cap K$.

To construct another hyperbolic line through p and parallel to ℓ, take a point x on $\mathbb{R}$ between K and L, and let A be the Euclidean circle centred on $\mathbb{R}$ that passes through x and p. We know that such a Euclidean circle A exists since $\mathrm{Re}(x) \neq \mathrm{Re}(p)$.

By construction, A is disjoint from L, and so the hyperbolic line $\mathbb{H} \cap A$ is disjoint from ℓ. That is, $\mathbb{H} \cap A$ is a second hyperbolic line through p that is parallel to ℓ. Since there are infinitely many points on $\mathbb{R}$ between K and L, this construction gives infinitely many different hyperbolic lines through p and parallel to ℓ. A picture of this phenomenon is given in Fig. 1.2.

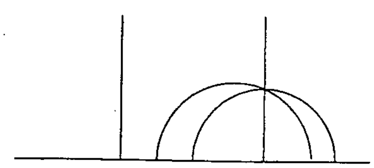

Figure 1.2: Several parallel hyperbolic lines

Exercise 1.4

Give an explicit description of two hyperbolic lines in $\mathbb{H}$ through i and parallel to the hyperbolic line $\ell = \mathbb{H} \cap \{z \in \mathbb{C} \mid \mathrm{Re}(z) = 3\}$.

Now, suppose that ℓ is contained in a Euclidean circle A. Let D be the Euclidean circle that is concentric to A and that passes through p. Since concentric circles are disjoint and have the same centre, one hyperbolic line through p and parallel to ℓ is the intersection $\mathbb{H} \cap D$.

To construct a second hyperbolic line through p and parallel to ℓ, take any point x on $\mathbb{R}$ between A and D. Let E be the Euclidean circle centred on $\mathbb{R}$ that passes through x and p. Again by construction, E and A are disjoint, and so $\mathbb{H} \cap E$ is a hyperbolic line through p parallel to ℓ.

As above, since there are infinitely many points on $\mathbb{R}$ between A and D, there are infinitely many hyperbolic lines through p parallel to ℓ. A picture of this phenomenon is given in Fig. 1.3.

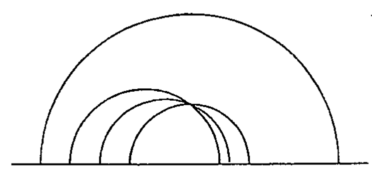

Figure 1.3: Several parallel hyperbolic lines

Exercise 1.5

Give an explicit description of two hyperbolic lines in $\mathbb{H}$ through i and parallel to the hyperbolic line $\ell = \mathbb{H} \cap A$, where A is the Euclidean circle with Euclidean centre -2 and Euclidean radius 1.

We now have a model to play with. The bulk of this book is spent exploring this particular model of the hyperbolic plane, though we do spend some time developing and exploring other models as well.

We close this section with a few words to put what we are doing in a historical context. We are proceeding almost completely backwards in our development of hyperbolic geometry from the historical development of the subject. A much more common approach is to begin with the axiomization of Euclidean geometry. One of the axioms is the statement about parallel lines mentioned above, namely that given a line Euclidean L and a point p not on L, there exists a unique Euclidean line through p and parallel to L. This axiom is often referred to as the Parallel Postulate; the form we give here is credited to Playfair.

Hyperbolic geometry is then defined by using the same set of axioms as Euclidean geometry, with the hyperbolic variant of the Parallel Postulate, namely that given a hyperbolic line ℓ and a point p not on ℓ, there are at least two hyperbolic lines through p and parallel to ℓ.

It is then shown that the upper half-plane model, with hyperbolic lines as we have defined them, is a model of the resulting non-Euclidean geometry. For instance, see the books of Stahl [24] and Greenberg [10], as well as the other sources mentioned in the list of Further Reading.

In this book, we are less concerned with the axiomatic approach to hyperbolic geometry, preferring to make use of the fact that we have reasonable coordinates in the upper half-plane $\mathbb{H}$, which allow us to calculate fairly directly.

Our first major task is to determine whether we have enough information in this definition of hyperbolic geometry to define the notions of hyperbolic length, hyperbolic distance, and hyperbolic area in $\mathbb{H}$. We do this using the group of transformations of $\mathbb{H}$ taking hyperbolic lines to hyperbolic lines.

1.2 The Riemann Sphere $\overline{\mathbb{C}}$

In order to determine the transformations of $\mathbb{H}$ that take hyperbolic lines to hyperbolic lines, we first fulfil our earlier promise of unifying the two seemingly different types of hyperbolic line, namely those contained in a Euclidean line and those contained in a Euclidean circle. We take as our stepping off point the observation that a Euclidean circle can be obtained from a Euclidean line by adding a single point.

To be explicit, let $\mathbb{S}^1$ be the unit circle in $\mathbb{C}$, and consider the function

$$\xi : \mathbb{S}^1 - \{i\} \to \mathbb{R}$$

defined as follows: given a point z in $\mathbb{S}^1 - \{i\}$, let K_z be the Euclidean line passing through i and z, and set $\xi(z) = \mathbb{R} \cap K_z$. This function is well-defined, since K_z and $\mathbb{R}$ intersect in a unique point as long as $\text{Im}(z) \neq 1$. See Fig. 1.4.

This operation is referred to as *stereographic projection*. In terms of the usual cartesian coordinates on the plane, the real axis $\mathbb{R}$ in $\mathbb{C}$ corresponds to the x-axis, and so $\xi(z)$ is the x-intercept of K_z. Calculating, we see that K_z has slope

$$m = \frac{\text{Im}(z) - 1}{\text{Re}(z)}$$

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